OBJECTIVES

On completion of this lesson, the student will be able to:

1. Recognize the series and parallel branches in series-parallel circuits.

2. Identify and arrange in correct order the steps for simplifying series-parallel circuits.

3. Determine equivalent resistance $R_{eq}$ in series-parallel circuits.

4. Analyze a series-parallel circuit and solve for unknown quantities of voltage and current.

5. Measure voltage, current, and resistance in series-parallel circuits and demonstrate the validity of calculated values by comparing them with the measured values.

PREREQUISITES

None

EQUIPMENT REQUIRED

Nida Model 130E Test Console
Nida Series 130 Experiment Card
PC130-9A
Nida Model 480/488 Multimeter, or equivalent

OVERVIEW

This lesson introduces students to the relationship of current, voltage, and resistance in series-parallel circuits.

From an analysis and discussion of series-parallel circuits, students learn to identify both series and parallel branches in the circuits.

By using techniques and equations learned in Block 3, Lessons 2 and 4, students learn to break down series-parallel circuits into simplified, equivalent circuits.

Students next learn to determine equivalent resistance. They then solve for unknown circuit values in series-parallel circuits.

The experiment provides students with hands-on practice in measuring voltage, current, and resistance in series-parallel circuits.

Students then compare their calculated values with their measured values. From this comparison, they see the proof of the relationship of current, voltage, and resistance in series-parallel circuits.
INTRODUCTION

In the last four lessons, you have learned how series circuits operate and how parallel circuits operate. Now you are ready to study a type of circuit that is a combination of those two circuits: the series-parallel circuit.

Some of you might be wondering why we have series-parallel circuits if we already have series circuits and parallel circuits. As stated in an earlier lesson, we use resistors to set voltages and currents to desired values. To do this, electronics engineers have to design circuits consisting of many components interconnected in many different ways.

Most electronic equipment you will deal with involves series-parallel circuits. Consider a typical radio circuit, for example, or your test console circuits, or even the circuit in your multimeter. You’ll find very few simple series or parallel circuits in equipment like this. In most cases, the circuits are combinations of both series and parallel circuits--they are series-parallel circuits.

You can use your knowledge of how series and parallel circuits operate to recognize and isolate the series and parallel branches in a series-parallel circuit. From this point, you will then be able to simplify and analyze series-parallel circuits.

Series-parallel circuits are really not all that hard to figure out. It’s really quite easy, too, if you take things step by step.

Let’s get started.

WHAT IS A SERIES-PARALLEL CIRCUIT?

How can you recognize a series-parallel circuit? That’s a good question, because you’ll find an unending variety of them.

Some of the components in a series-parallel circuit will be connected in series (a series branch) and some will be connected in parallel (a parallel branch). These series and parallel branches can be connected again in series or in parallel, making new series and parallel branches. These new series and parallel branches can be connected again in series or in parallel, and so on, and on, and on.

To learn about series-parallel circuits, perhaps we’d better start with the definition.

DEFINITION

SERIES-PARALLEL CIRCUIT: An electronic circuit consisting of a group of series and parallel components where at least one component lies in the path of the total current.

An even better way of explaining series-parallel circuits is to show you some typical examples. In Figure 1, we’ve drawn the circuit diagrams of six of the simpler series-parallel circuit configurations.
The circuit in Figure 1A contains three resistors, R1, R2, and R3. You can see readily that R2 and R3 are in parallel. We can replace these two resistors with one total equivalent resistor, which then is in series with R1.

The circuit in Figure 1B contains four resistors, R1, R2, R3, and R4. R3 and R4 are in series. At the same time, R3 and R4 are also in parallel with R2. In addition, the parallel circuit of R2 and R3 plus R4 is in series with R1.

The circuit in Figure 1C is an easy one. R1 and R2 are in series. R3 and R4 are also in series. If you replace each series branch with its total equivalent resistance, the total resistances are then in parallel.

The circuit in Figure 1D is also an easy one. R2 and R3 are in parallel, and their total equivalent resistance is in series with R1 and R4.

The circuit in Figure 1E looks more complex, but check again. It really is quite simple. R1 and R2 are in parallel, R3 and R4 are in series, and R5 is in parallel with R3 plus R4.

If you replace both R1 and R2 and R3 and R4 with their total equivalent resistances, you’ll find that you have a circuit just like the one in Figure 1A.

The circuit in Figure 1F is one you will see frequently. R5 and R6, which are in series, are in parallel with R4. This combination is in series with R3, and the total combination, in turn, is in parallel with R2. This gives us, once again, the circuit of Figure 1A.
Look closely at these circuits. Notice that as we add more resistors to the circuits, we can arrange the circuits in increasingly complex ways. All of the circuits, however, are series-parallel circuits. As you might guess, analyzing these circuits can prove quite time-consuming.

**EQUIVALENT RESISTANCE**

What is equivalent resistance? Maybe some of you can guess what it is after reading the first part of this lesson, but here’s the definition anyway.

**DEFINITION**

**EQUIVALENT RESISTANCE:** The calculated total resistance of the series and parallel branches in a series-parallel circuit.

You know how to calculate the total resistance ($R_T$) in both series circuits and parallel circuits.

In a series-parallel circuit, you add the equivalent resistance to produce $R_T$. We even have an abbreviation for equivalent resistance, just like we do for total resistance. The abbreviation is $R_{eq}$.

You’re probably thinking that calculating $R_T$ is going to be pretty hard to do, considering how complex some series-parallel circuits can be. Actually, it’s not as difficult as you might think, but it often does take a lot of time.

The secret behind calculating $R_T$ in series-parallel circuits is that you must first simplify or break down the series-parallel circuit into its separate series and parallel branches. Then you can calculate individual equivalent resistances in the separate branches of the circuit. Once you learn to do this, calculating $R_T$ in a series-parallel circuit is really pretty easy.

We’ve divided the simplifying procedure into the following four steps:

**STEP 1:** IDENTIFY THE SERIES AND PARALLEL BRANCHES in the circuit.

**STEP 2:** CALCULATE THE EQUIVALENT RESISTANCE for each series and parallel branch of the circuit.

**STEP 3:** REDRAW THE CIRCUIT with the calculated equivalent resistances replacing the series and parallel branches of the circuit.

**STEP 4:** SIMPLIFY THE CIRCUIT FURTHER by repeating Steps 1, 2, and 3 until you have only one total resistance left--$R_T$.

We’ll use these steps with a real series-parallel circuit to explain how they work. To do this, we need to have a diagram of a series-parallel circuit, one for which we know the basic circuit values.

The circuit of Figure 1A is a good one to start with. First, however, we’ll assign some values to the circuit, which we have done in Figure 2. Now we can simplify this series-parallel circuit and calculate $R_T$, starting with Step 1.
**STEP 1: IDENTIFY THE SERIES AND PARALLEL BRANCHES.**

Look at the series-parallel circuit in the diagram of Figure 2.

Identify the series and/or parallel branches in this circuit.

Since this is a very simple series-parallel circuit, we have only two branches, one with R2 and R3 in parallel and one with R2 and R3 in series with R1.

Now wasn’t that easy?

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**STEP 2: CALCULATE THE TOTAL RESISTANCE.**

This should be easy too, since you already know how to calculate the total resistance in a parallel circuit using the formula:

Formula for solving for $R_{eq}$:

$$R_{eq} = \frac{R_2 \times R_3}{R_2 + R_3}$$

Substitute circuit values:

$$R_{eq} = \frac{12 \Omega \times 12 \Omega}{12 \Omega + 12 \Omega} = \frac{144 \Omega^2}{24 \Omega} = 6 \Omega$$

Thus, $R_{eq}$ for the parallel branch containing R2 and R3 equals 6Ω.

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**STEP 3: REDRAW THE CIRCUIT.**

Now look at the simplified series-parallel circuit, as redrawn in the diagram of Figure 3.

This circuit is the same as the one in Figure 2, except for one thing. We now have $R_{eq}$ for the parallel branch instead of R2 and R3.
**STEP 4: SIMPLIFY THE CIRCUIT FURTHER.**

Now we’ll repeat the first three steps to simplify the circuit further.

**Step 1:** As you can see, you now have a simple series circuit consisting of R₁ and Req in series.

**Step 2:** You know the formula for calculating \( R_T \) in series circuits: \( R_T = R_1 + R_2 + \ldots \).

As you know, \( R_T \) is the total resistance in a series-parallel circuit. Since you now have a simple series circuit, you can use the formula above. Therefore, let’s solve for \( R_T \) in the circuit of Figure 3.

First, we need the formula: \( R_T = R_1 + R_{eq} \)

Substitute the circuit values: \( R_T = 6 \, \Omega + 6 \, \Omega = 12 \, \Omega \)

**Step 3:** Look at the simplified circuit, as redrawn in Figure 4.

As you can see, we now have a total resistance of 12 Ω in the simplified series-parallel circuit, which is \( R_T \).

Here’s an example using the circuit from Figure 1C. Working through the example step by step will explain further how to simplify and analyze series-parallel circuits and calculate \( R_T \).

**Example 1: Using the Four Steps to Calculate Equivalent Resistance.**

We’ll use the series-parallel circuit in the diagram of Figure 5, with circuit values as indicated.

**Step 1:** Identify the series and parallel branches.

The circuit has two branches in series: Branch 1, with R₁ and R₂ in series; and Branch 2, with R₃ and R₄ in series.
Step 2: Calculate the equivalent resistance by solving for $R_{eq1}$ in Branch 1 and $R_{eq2}$ in Branch 2.

Equivalent resistance of Branch 1: $R_{eq1} = R1 + R2 = 15\,\Omega + 5\,\Omega = 20\,\Omega$

Equivalent resistance of Branch 2: $R_{eq2} = R3 + R4 = 8\,\Omega + 12\,\Omega = 20\,\Omega$

Step 3: You can now redraw this circuit as shown in Figure 6.

![Figure 6. Step 3](image)

Step 4: Now simplify further the simple parallel circuit in Figure 6 (repeat Step 1).

Next solve for the total resistance of this circuit. This gives you $R_T$ of the original series-parallel circuit in Figure 5 (repeat Step 2).

Formula for solving for $R_T$: $R_T = \frac{R_{eq1} \times R_{eq2}}{R_{eq1} + R_{eq2}}$

Substitute circuit values: $R_T = \frac{20\,\Omega \times 20\,\Omega}{20\,\Omega + 20\,\Omega} = \frac{400\,\Omega^2}{40\,\Omega} = 10\,\Omega$

Thus, $R_T$ of the series-parallel circuit in Figure 5 equals 10\,\Omega.

The simplified circuit is redrawn showing $R_T$ in Figure 7 (repeat Step 3).

![Figure 7. Step 4](image)

Since we have determined $R_T$, we have simplified the circuit as far as we can.
Exercise 1: Calculating Total Resistance.

Now let’s see if you can calculate the total resistance of the circuits in the problems below. Show your calculations in the space provided.

1. Given the series-parallel circuit of Figure 8, find the total resistance $R_T$.

$$R_T = \quad \quad$$

![Figure 8. Problem 1](image)

2. Given the series-parallel circuit of Figure 9, find the total resistance $R_T$.

$$R_T = \quad \quad$$

![Figure 9. Problem 2](image)
CURRENT AND VOLTAGE

When you analyze a typical electronic circuit, you will usually know the values for the components and the applied voltage. The quantities you most frequently need to find are the total current being drawn from the power source and the voltage at any given point in the circuit.

Calculating total current $I_T$ is easy if you know applied voltage $E_a$ and can find the total resistance using the formula $I_T = \frac{E_a}{R_T}$.

To find the voltage at a certain point in the circuit, you must know how to do two things: determine voltage drops in series circuits and solve for branch currents in parallel circuits. Do you remember these rules?

- ♦ The sum of the voltage drops across resistors in a series circuit equals the voltage applied across the resistors.
- ♦ The total current in a parallel circuit equals the sum of all the branch currents.

Let’s run through an example to show you how to solve for current and voltage in series-parallel circuits.

Example 2: Solving for Current and Voltage.

Consider the series-parallel circuit in the diagram in Figure 10. The circuit has four resistors and an applied voltage of 15 V. TP4 is ground. The diagram includes all resistor values. What you must do is find the voltage at TP3.

What you know:

- $E_a = 15$ V
- $R_1 = 1$ kΩ
- $R_2 = 2$ kΩ
- $R_3 = 2$ kΩ
- $R_4 = 1$ kΩ

What you don’t know:

- $I_1 = ?$
- $I_2 = ?$
- $I_T = ?$

To find the voltage at TP3, you need to find $E_{R4}$, the voltage drop across $R_4$. Ohm’s Law tells you that $E_{R4} = I_T \times R_4$. To find $E_{R4}$, therefore, you must first solve for $I_T$. Fortunately, you know that $I_T$ equals the applied voltage divided by the total resistance of the circuit, expressed in the formula $I_T = \frac{E_a}{R_T}$. You already know $E_a$, so all you need to do is find $R_T$. 

Figure 10. Solve for Voltage at TP3
First, determine that resistors R2 and R3 are in parallel (Step 1). Next, simplify the circuit by reducing R2 and R3 to $R_{eq}$ (Step 2).

$$R_{eq} = \frac{R_2 \times R_3}{R_2 + R_3} = \frac{2000 \ \Omega \times 2000 \ \Omega}{2000 \ \Omega + 2000 \ \Omega} = \frac{4,000,000 \ \Omega}{4,000} = 1,000 \ \Omega = 1 \ k\Omega$$

Then, redraw the circuit, as shown in Figure 11 (Step 3). Now you can solve for $I_T$.

First, solve for $R_T$:

$$R_T = R_1 + R_{eq} + R_4$$
$$R_T = 1 \ k\Omega + 1 \ k\Omega + 1 \ k\Omega$$
$$R_T = 3 \ k\Omega$$

Therefore:

$$I_T = \frac{E_a}{R_T} = \frac{15 \ V}{3000 \ \Omega} = .005 \ A \ or \ 5 \ mA$$

Now that you know $I_T$, you can determine $E_{R4}$.

Calculate the voltage drop:

$$E_{R4} = I_T \times R_4 = .005 \ A \times 1000 \ \Omega = 5 \ V$$

Solve for voltage at TP3:

$$E_{TP3} = E_{R4} = 5 \ V$$

Here's another example to give you some more practice.

**Example 3: Solving for Current and Voltage.**

Given the circuit in Figure 12, you want to find $I_2$.

What you know:

- $E_a = 12 \ V$
- $R_1 = 100 \ \Omega$
- $R_2 = 220 \ \Omega$
- $R_3 = 380 \ \Omega$
- $R_4 = 620 \ \Omega$
- $R_5 = 180 \ \Omega$

What you want to find:

$I_2 = ?$
You can see at a glance that we have a series-parallel circuit with two branches. Branch 1 has resistors R2 and R3 in series. Branch 2 has resistors R4 and R5 in series.

To find Branch 2 current $I_2$, we have two choices. We can determine total current $I_T$ and Branch 1 current $I_1$, since $I_T = I_1 + I_2$, or we can determine $I_T$ and then find the voltage at TP2. Once we know $E_{TP2}$, we can determine $I_2$. This method is the one we will use.

To calculate $I_T$, we need to know $R_T$, so first we must simplify the circuit by finding $R_{eq1}$ and $R_{eq2}$.

\[
R_{eq1} = R_2 + R_3 = 220 \, \Omega + 380 \, \Omega = 600 \, \Omega
\]

\[
R_{eq2} = R_4 + R_5 = 620 \, \Omega + 180 \, \Omega = 800 \, \Omega
\]

Now we can draw the circuit shown in Figure 13 which, as you can see, is a simple parallel circuit. We can simplify this circuit further by finding equivalent resistance $R_{eq1}$ and $R_{eq2}$ or $R_{eq3}$.

![Figure 13. Solve for Total Resistance](image)

We know that:

\[
R_{eq3} = \frac{R_{eq1} \times R_{eq2}}{R_{eq1} + R_{eq2}}
\]

\[
R_{eq3} = \frac{600 \, \Omega \times 800 \, \Omega}{600 \, \Omega + 800 \, \Omega} = \frac{480,000 \, \Omega}{1400}
\]

Therefore:

\[
R_{eq3} = 342.8 \, \Omega
\]

Now we have a simple series circuit, so we can find $R_T$, which we need in order to determine $I_T$.

\[
R_T = R_1 + R_{eq3} = 100 \, \Omega + 342.8 \, \Omega = 442.8 \, \Omega
\]

Using Ohm’s Law, we now solve for $I_T$.

\[
I_T = \frac{E_a}{R_T} = \frac{12 \, V}{442.8 \, \Omega} = 0.0271 \, A \text{ or } 27.1 \, mA
\]

Now that we know $I_T$, we must find the voltage across TP1 and TP2, which is the voltage drop across R1, $E_{R1}$.

Use this formula:

\[
E_{R1} = I_T \times R_1
\]

Therefore:

\[
E_{R1} = 0.0271 \, A \times 100 \, \Omega = 2.71 \, V
\]
Next we must find $E_{\text{Req2}}$, the voltage between TP2 and TP3, which is the voltage drop across $R_{\text{eq2}}$. As we determined earlier, $R_{\text{eq2}}$ equals $800 \, \Omega$.

Use this formula: 

$$E_{\text{Req2}} = I_2 \times R_{\text{eq2}}$$

Substitute known values: 

$$E_{\text{Req2}} = I_2 \times 800 \, \Omega = I_2 \times (800 \, \Omega)$$

By substituting known values in the formula $E_a = E_{R1} + E_{R2} + \ldots$, we can now solve for the voltage at TP2. Remember: in a series circuit, the applied voltage equals the sum of the voltage drops around the circuit. Thus the applied voltage is equal to the voltage drop across TP1 and TP2 plus the voltage drop across TP2 and TP3.

Use this formula: 

$$E_a = E_{R1} + E_{\text{Req2}} \quad \text{so:} \quad E_{\text{Req2}} = E_a - E_{R1}$$

Substitute known values: 

$$I_2 \times (800 \, \Omega) = 12 \, V - 2.71 \, V = 9.29 \, V$$

Therefore:

$$I_2 \times (800 \, \Omega) = 9.29 \, V \quad \text{so:} \quad I_2 = \frac{9.29 \, V}{800 \, \Omega} = .0116 \, A$$

We have solved the problem: 

$$I_2 = 0.116 \, A = 11.6 \, mA$$

**Exercise 2: Solving for Current and Voltage.**

Now you'll get a chance to solve a some problems on your own. Determine the circuit values, as requested in each problem.

1. Given the circuit in Figure 14, find the voltage drop across R2.

   $E_{R2} = \phantom{000}$

   ![Circuit Diagram](image)

   You know that:
   
   $E_a = 20 \, V$

   $R1 = 150 \, \Omega$

   $R2 = 100 \, \Omega$

   $R3 = 100 \, \Omega$

   **Figure 14. Problem 1**
Exercise 2: (continued)

2. Given the circuit in Figure 15, find the voltage drop across R5.

\[ E_{R5} = \] __________

You know that:
\[ E_a = 10 \text{ V} \]
\[ R1 = 2 \text{ k}\Omega \]
\[ R2 = 2 \text{ k}\Omega \]
\[ R3 = 4 \text{ k}\Omega \]
\[ R4 = 12 \text{ k}\Omega \]
\[ R5 = 12 \text{ k}\Omega \]

![Figure 15. Problem 2](image1.png)

3. Given the circuit in Figure 16, find the current flowing through R5.

\[ I_{R5} = \] __________

You know that:
\[ E_A = 25 \text{ V} \]
\[ R1 = 200 \text{ \Omega} \]
\[ R2 = 1 \text{ k}\Omega \]
\[ R3 = 1.8 \text{ k}\Omega \]
\[ R4 = 2 \text{ k}\Omega \]
\[ R5 = 1 \text{ k}\Omega \]
\[ R6 = 1 \text{ k}\Omega \]

![Figure 16. Problem 3](image2.png)
SUMMARY

This lesson has introduced you to series-parallel circuits. Here are the important points for you to remember.

♦ Most circuits you will see in electronics are series-parallel circuits.

♦ You can simplify any series-parallel circuit branch that contains resistors in series by adding the series resistors to determine one equivalent resistance, $R_{eq}$.

♦ You can replace any series-parallel circuit branches that are in parallel with one branch that has one equivalent resistance, $R_{eq}$.

♦ By continuing to combine parallel and series branches, you can further simplify any series-parallel circuit until it contains only one total resistance, $R_T$.

♦ Given the applied voltage and the resistor values in a series-parallel circuit, you can analyze the circuit and determine all currents and voltage drops.
  - You determine total current drawn by using the total resistance of the circuit.
  - You can next determine voltage drops by using the value of the total current in the circuit.
  - You can then determine the currents of all parallel branches by using the voltage drop values.